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Recent Progress in Detonation Research

A. K. Oppenheim
University of California, Berkeley, Calif.

N. Manson
University of Poitiers, France

and

H. Gg. Wagner
University of Göttingen, Germany

THE major effort of currently conducted detonation studies is concerned mainly with three fundamental problems: the development of the wave, its stability, and structure. The three are of course not independent of each other, and even the order of presentation is of some importance, for one cannot describe the structure without considering the stability, which, in turn, is related to the mode of development. Consequently this review consists of three corresponding parts, each written, respectively, by one of the authors in a different country, thus giving the subject matter an elucidation from three quite different points of view. Because of its particular significance to the process of combustion in a rocket thrust chamber, the scope is restricted to the gaseous phase.

With respect to the development, most significant were the studies of the generation of pressure waves by accelerating flames and the observations of the onset of transverse oscillations at the instant when the retonation wave originates.

As far as the stability is concerned, it appears that small-amplitude, high-frequency oscillations of the wave are almost

A. K. Oppenheim, Professor of Aeronautical Sciences, studied aeronautical engineering at the Warsaw Institute of Technology and received the Dipl. Ing. in 1943. He obtained his Ph.D. (University of London) and D.I.C. (Imperial College) in 1945. He served on faculties of City and Guilds College (London), 1945–1948, and Stanford University, 1948–1950. Since 1950 he has been at the University of California, Berkeley. The 1960–1961 academic year he spent in Paris as a visiting Professor at the Sorbonne, on Senior Postgraduate Fellowship of the National Science Foundation. He is a Member of AIAA.

N. Manson has worked in the field of applied thermodynamics, especially high-temperature properties as well as shock and combustion wave phenomena, since 1935. He started as a Research Engineer in various French research establishments and obtained his degree of Doctor of Physical Sciences at the University of Paris in 1946. Nominated as a Professor at the University of Poitiers in 1954, he taught thermodynamics of propulsion and of high temperatures and conducted a research laboratory associated with these subjects. In 1959, the Academy of Sciences of Paris conferred upon him a prize for his work on shock and detonation waves. He is a member of the Combustion Institute and President of Poitiers Section of the French Society of Astronautics. Since 1962 he has been the Director of the Ecole Nationale Superieure de Mecanique et d'Aerotechnique, the school of engineering at the University of Poitiers.

H. Gg. Wagner received his first diploma at the University of Darmstadt in 1953 and the degree of Doctor Rerus Naturalis at the same university in 1956. In most of his scientific work he was associated with Wilhelm Jost at the University of Göttingen, where, in 1960, he was nominated to the position of a Docent in Physical Chemistry. In 1961, he was a Visiting Lecturer of the American Institute of Chemical Engineers. His major contributions are in the field of reaction kinetics and combustion phenomena.

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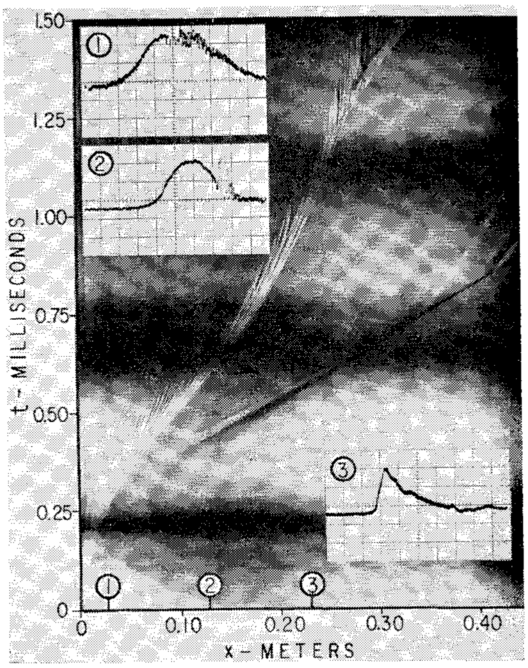

Fig. 1 Streak schlieren photograph of the development of detonation in stoichiometric hydrogen-oxygen mixture initially at normal temperature and pressure showing the generation of pressure waves ahead of the accelerating flame. Spark ignition by discharging 1.0 mjoule across $\frac{1}{32}$-in. gap. Electrodes located at closed end of a 1- × $1\frac{1}{2}$-in. cross-section tube. Pressure records at positions 1, 2, and 3 shown as inserts. Vertical grid: 5.2 psi/div for insert 1, 10.4 psi/div for inserts 2 and 3. Horizontal grid: 0.10 msec/div for inserts 1 and 2, 0.20 msec/div for insert 3, from left to right.[48]

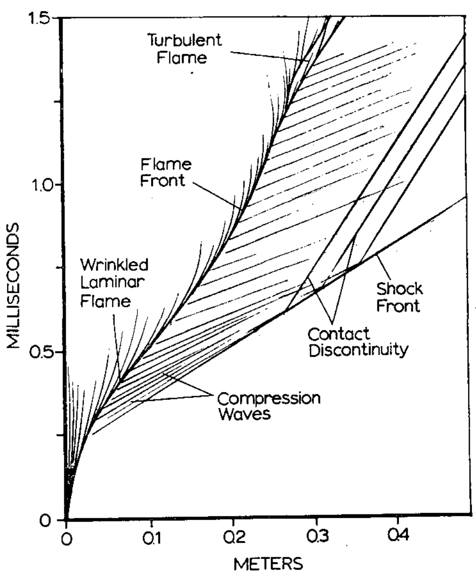

Fig. 2 Interpretation of the streak photograph of Fig. 1.

universally apparent, whereas the low-frequency instability is associated with an actual separation of the combustion zone from the shock front. Evidence also has been produced to the effect that the nonsteady wave manifests itself often in the form of a multidimensional configuration.

Studies of the structure provided an insight into the kinetics of chemical reactions under the nonequilibrium conditions of significant pressure and temperature gradients. The evidence at hand seems to favor the concept of the von Neumann-Döring-Zeldovich theory, according to which the detonation consists of two distinct wave fronts: the shock and the deflagration. It is doubtful, however, whether the wave process can be regarded as a sequence of equilibrium states. Furthermore, at least under one set of operating conditions, the wave has been found to be turbulent in character.

The importance of detonation research to rocket technology stems from the fact that, besides the possibility of its direct exploitation, the detonation wave produces conditions resembling quite closely those prevalent in high-performance thrust chambers so that it can serve as a convenient means for the study of their chemicokinetic and gas-wave-dynamic processes. Of particular interest in this respect is the relationship between the kinetics of the combustion reaction and the dynamics of pressure waves that are formed when heat is released at a high power-density level. In fact, the subject of detonation research can be regarded as one concerning, in

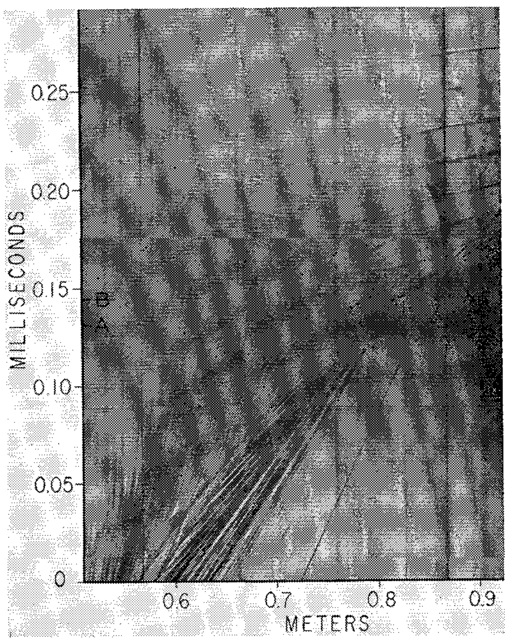

Fig. 3 Streak schlieren photograph of the onset of retonation in a stoichiometric hydrogen-oxygen mixture initially at normal temperature and pressure. Hot-wire ignitor, made up of a $\frac{1}{2}$-in.-long by $\frac{1}{8}$-in.-diam electrically heated coil, located at closed end of 1- × 1-$\frac{1}{2}$-in.-cross section tube. The abscissa scale denotes the distance from the end of the tube. Symbols *A* and *B* mark instances at which the accompanying flash schlieren phorographs have been obtained.[68]

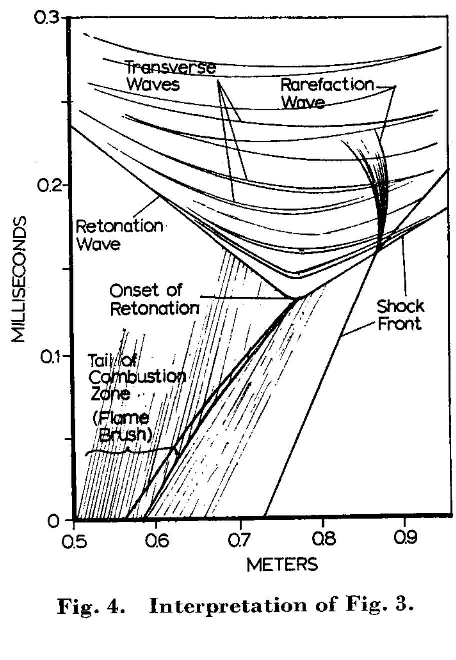

Fig. 4. Interpretation of Fig. 3.

general, the whole field of combustion phenomena that are associated implicity with pressure waves. Among these, the instability effects play an especially significant role.

In view of the excellent prospective that has been attained as a result of a number of comprehensive publications which became available recently, the main emphasis in this paper is on the progress made over the past two years. For background information, which, incidentally, is not required for the understanding of this review, the reader is referred to the papers of Evans and Ablow,[30] Fay,[36] Gross,[40] Oppenheim,[40, 67] Wagner,[96, 97] and Manson,[54] to the texts of Zeldovich and Kompaneets,[107] of Sokolik,[79] and of Salamandra et al.,[75] and to the interesting collection of reports on the work performed over a number of years in the Combustion Physics Laboratory of the Institute of Energetics of the USSR Academy of Sciences.[70] For more advanced treatment of the gasdynamic aspects, one should not overlook Chaps. VII–X of the remarkable book of Stanyukovich.[85]

Development

As a salient feature of the nonlinear character of the detonation process, many of its properties, notably its stability and structure, depend on the mode of initiation. For the sake of clarity, it is therefore most convenient to start the exposition with the consideration of the development of the wave.

The extent of the transient processes that culminate in the formation of the steady wave can be expressed in terms of a single over-all parameter, the so-called induction distance, that is, the length of the run of the flame from the ignition source to a point where the detonation wave is ostentatiously established. The latter, as used by most investigators, is, in fact, the point of the onset of the retonation wave. In this respect, Bollinger and Edse[6–8] added to the wealth of data they have already amassed and proposed a theoretical rationalization for their measurements. In more recent publications they described specifically the influence of the tube diameter, the ignition source, and various geometrical effects, such as the variation of the cross section, right-angle bends, and obstacles in the detonation tube.

The effect of the tube size was also studied photographically by Baumann, Urtiew, and Oppenheim,[4] whereas that of turbulence in a flowing medium was studied by Baumann and Wagner.[3] Interestingly enough, both investigations yielded a similar number for the limiting value of the ratio of the induction distance to induction time (induction "velocity" of 0.65 km/sec) which was reached with quite different gas mixtures when the Reynolds number became sufficiently high to render the influence of walls ineffective.

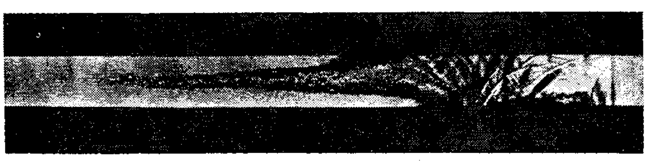

Fig. 5 Flash schlieren photograph of the onset of retonation in a stoichiometric hydrogen-oxygen mixture initially at normal temperature and pressure at an instant marked by *A* on the streak schlieren photograph (Fig. 3). Field of view 75 cm from hot-wire ignitor across the full 1-in. width of its 1- × 1½-in. cross section.[68]

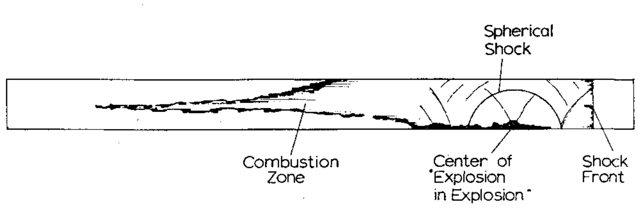

Fig. 6 Interpretation of Fig. 5.

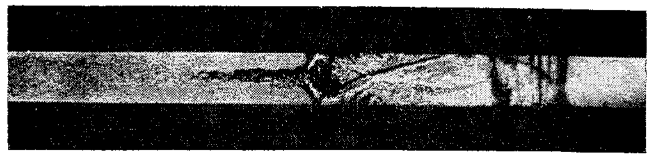

Fig. 7 Flash schlieren photograph of transverse waves set up at the onset of retonation in a stoichiometric hydrogen-oyxgen mixture initially at normal temperature and pressure shown at an instant marked by *B* on the streak schlieren photograph (Fig. 3). Field of view 75 cm from hot-wire ignitor across the full 1-in. width of its 1- × 1½-in. cross section.[68]

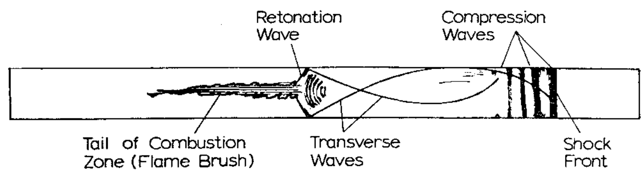

Fig. 8 Interpretation of Fig. 7.

Of particular interest to the whole subject of detonation phenomena are the investigations concerned with the effect of various additives on the development of the wave and the study of detonability in the vicinity of composition limits, as exemplified by the studies of Miles, Munday, and Ubbelohde,[57] Seamans and Wolfhard,[77] Belles and Ehlers,[5] and Luker et al.[1, 53]

As contrasted to the over-all aspects, the detailed mechanism of the development of the detonation depends primarily on the action of pressure waves that are generated in the course of the process. This point has been given indeed a very special attention by the Russian investigators, as evidenced by the numerous publications of Salamandra et al.[70, 75] and Soloukhin,[80–83] as well as the papers of Kogarko,[48, 49] Novikov,[48, 66] Riazantsev,[66, 73] Babkin and Kozachenko,[2] and Gvozdeva,[41] to mention just a few.

Wave phenomena leading to the development of detonation can be described in terms of records that have been obtained recently by Laderman, Urtiew, and Oppenheim[50, 51, 67, 68] as follows.

Figures 1 and 2 demonstrate how a pressure wave is generated ahead of an accelerating flame at the initial stage of the process. In the particular case of this record, the ignition was obtained by a spark that was sufficiently weak so that no disturbance was introduced into the unburned medium. The figures show that a shock front is generated about 0.5 msec after ignition ahead of a laminar flame, thus providing an experimental proof that a shock front can be generated by an accelerating flame before it breaks up into a turbulent brush.

Figures 3 and 4 show how in the later stages of the process there occurs "an explosion in an explosion," a point explosion somewhere within the turbulent reaction zone, producing two strong shocks, one moving into the unburned medium, referred to as super-detonation, and the other into the burned gases, known as the retonation. Between the two there is set up a mode of transverse oscillations which has been previously referred to as "spin."

Figures 5 and 6 demonstrate how the "explosion in the explosion" is initiated, giving a strong indication that it takes the form of a spherical shock with a center located in the vicinity of the boundary layer. Finally, Figs. 7 and 8 show the transverse waves that are set in the reaction zone.

To sum up, the way it appears now, the development of detonation in a tube proceeds according to the following sequence of events. First a shock is formed ahead of an accelerating, laminar flame, which at this stage of the process has a cellular structure, giving it a character of a wrinkled combustion front. As a result of the motion induced by the shock, the flame breaks up into a turbulent brush. The point

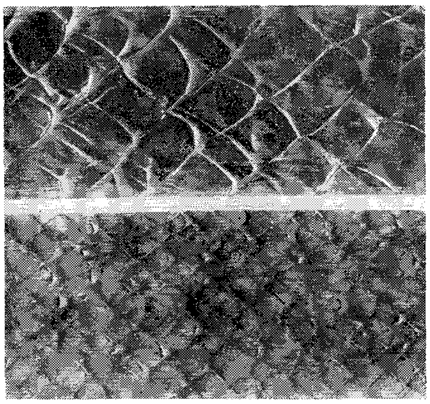

Fig. 9 Examples of interaction patterns obtained by Duff[25] with the use of the soot technique. Each shows the unfolded record taken around the full perifery of the tube. The upper represents a two- by three-headed spin, and the lower a nine-headed spin. The waves move from left to right.

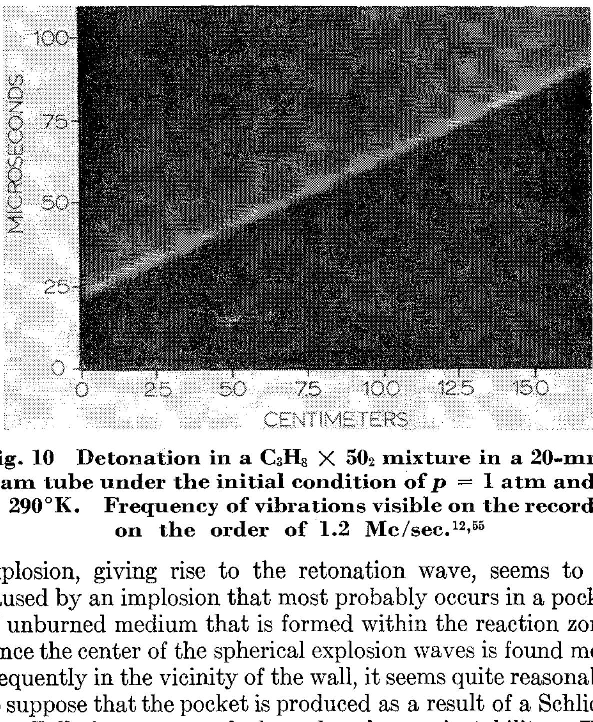

Fig. 10 Detonation in a $C_3H_8 \times 5O_2$ mixture in a 20-mm-diam tube under the initial condition of $p = 1$ atm and $T = 290°K$. Frequency of vibrations visible on the record is on the order of 1.2 Mc/sec.[12,55]

explosion, giving rise to the retonation wave, seems to be caused by an implosion that most probably occurs in a pocket of unburned medium that is formed within the reaction zone. Since the center of the spherical explosion waves is found most frequently in the vicinity of the wall, it seems quite reasonable to suppose that the pocket is produced as a result of a Schlichting-Tollmien type of boundary-layer instability. The "spin" that is triggered in this manner is evidently controlled by the kinetics of chemical reactions, since its amplitude is large near the detonation limits where the over-all rate of chemical reaction is slow, whereas it becomes attenuated close to the stoichiometric composition where the rate attains its maximum value. The final "steady wave" is established as a result of a long sequence of wave interaction processes that lead finally to the shock-deflagration ensemble: the self-sustained or Chapman-Jouguet detonation.

Stability

The problem of primary interest with respect to stability is that concerned with self-sustained rather than overdriven detonations. Recent studies of such waves, as for instance those carried out by Manson and his associates,[9, 11, 54, 55] yielded quite a definite conclusion to the effect that, in general, there exists a small (from 0.3 to 1%) but systematic deviation of the propagation velocity D_∞ (deduced from measured velocities D_d in long tubes of different diameters d) from the value $D_{CJ}°$ calculated according to the classical Chapman-Jouguet theory with the use of the best available thermodynamic data under the assumption that the substance behaves essentially as a mixture of perfect gases with variable equilibrium composition (see, e.g., Foreman et al.[38]).

For certain mixtures, a plausible explanation for this deviation can be obtained by attributing it either to an uncertainty in some numerical data used for computations or to a departure in the behavior of the burned gas from that of a perfect gas mixture.[54] There are also, however, numerous instances where the deviation can be explained only by effects that have not been considered in the Chapman-Jouguet theory, that is, by those whose influence cannot be completely eliminated by the evaluation of D_∞ by a linear extrapolation of the measured $D_d(1/d)$ to $1/d = 0$. Among such phenomena, the spinning detonation became especially significant in that its study yielded recently novel experimental techniques that provided also a better insight into the structure of the detonation wave. Besides revealing some interesting properties of the relationship between the shock wave and the reaction zone, which will be discussed in the next section, they enhanced principally, of course, our understanding of the stability.

Most of the experiments were performed with mixtures of hydrogen and hydrocarbons with oxygen and nitrogen, initially at normal and subatmospheric pressures, using either a new technique based on the observation of traces left by the wave on a soot-coated wall, which has been introduced by Denisov, Troshin et al.,[16–20, 94] in the USSR and applied by Duff[25] in the United States, or by various optical methods of observation such as the schlieren system, exemplified by the work of Edwards et al.,[27–29] Fay,[34, 35] Opel,[34] Manson et al.,[9–12, 55] Oppenheim et al.,[68] Wagner et al.,[21, 45] Voitsekhovsky et al.,[89–93] and Mitrofanov,[58] or by the interferometer technique that has been employed so impressively by White.[56, 100, 101] In spite of some differences in the interpretation, all of these experiments yielded one general conclusion, namely that, even quite far from the walls, the motion of the gas in the vicinity of the reaction zone, and probably also in the vicinity of the shock wave, is never rigorously one-dimensional. In fact, depending on the specific experimental method of observation, the flow appeared as one associated either with a multitude of inflammation "heads," as illustrated by Fig. 9, or with some high-frequency (10^2 to 10^3 kc/sec) oscillations, as shown in Fig. 10, or, finally, with turbulence, as indicated in Fig. 11.

A variation in any of the parameters, such as the composition of the mixture (in terms of its departure from the stoi-

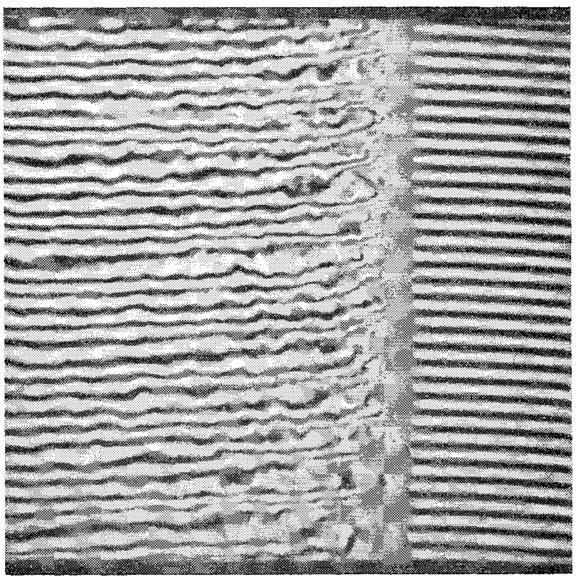

Fig. 11 Interferogram obtained by White[100] of a self-sustained detonation in a $2H_2 + O_2 + 2CO$ mixture initially at 0.3-atm pressure and room temperature. Increase in density shifts fringes upwards.

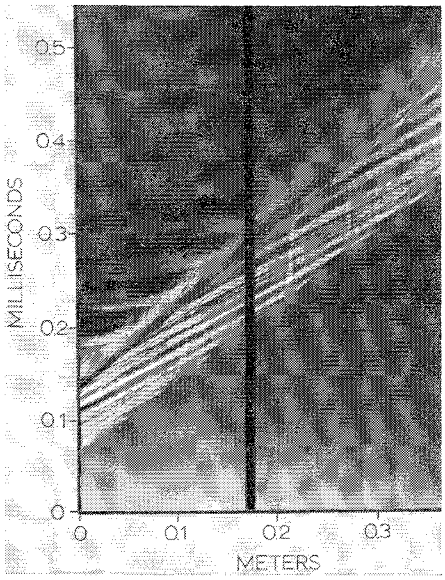

Fig. 12 Unstable detonation in a $C_3H_8 + 5O_2 + 16N_2$ mixture in a 20-mm-diam tube initially at $p = 1$ atm and $T = 290°K$. Between the shock and the reaction zone the gas is evidently in a state of vibration.[9,55]

chiometric ratio, or of the concentration of inert additives), its initial pressure, or the diameter of the tube, causes a variation in the amplitude of oscillations propagating at right angles to the direction of motion of the detonation wave. This deviation, apparently small in the case of a well-established, self-sustained detonation (that is, one propagating at a constant velocity D_a, which, within the precision of the measurement, is recorded with a deviation not exceeding ±0.2% over a length of at least 10 m[10, 12]), increases progressively as the limits of detonation are approached in either composition, initial pressure, or diameter. The detonation tends then to acquire a "spinning" mode, and it becomes finally quite unstable, so that its velocity of propagation varies significantly—in some instances by even more than 100%.

Schlieren observations of such unstable waves demonstrate that they are associated with an actual separation of the shock wave from the reaction zone, followed by the re-establishment of detonation which is carried out apparently in a manner quite similar to its initial onset from deflagration.[11] The experimental records reveal in particular the appearance of oblique parallel striations in the time-space plane, as shown in Fig. 12, where they are quite evident in the zone between the shock and the flame. These are interpreted as manifestations of the tri-dimensional oscillations that are sustained by the transverse waves in the burned gas behind the reaction zone.[39]

All this suggests that the oscillations serve as means for the transfer of energy from the reaction zone to the shock wave, thus providing an important, if not a determinant, effect upon the sustenance of the latter by the former. Since even far from the limits one observes only the change in the amplitude of these phenomena, either in terms of the change in frequency of vibrations or in the number of cells left on the soot-coated wall, one is led to the conclusion that *all self-sustained detonations are intrinsically unstable.*

The fundamental reason for the instability of the structure is not yet understood to our complete satisfaction. There are a number of hypotheses that have been proposed recently to interpret in a more-or-less comprehensive manner the various aspects of the observed unstable phenomena, such as the possible structure of the heads of inflammation put forward by Denisov and Troshin,[18] the effect of oscillations pointed out by Manson et al.[9, 11, 55] and by Guenoche,[39] and the influence of "turbulence" emphasized by White.[100] In the meantime it appears that, as long as the exact mechanism of energy transfer between the shock wave and the reaction zone, and consequently the detailed structure of the detonation wave, is not better known, it would be difficult to deduce a completely satisfactory theory for its stability.

Most interesting to date have been the mathematical investigations of the wave stability and structure which have been carried out by Wood and his co-workers.[103–105] Erpenbeck,[31–33] in particular, concentrated his attention on the so-called "square wave" model, described by Fig. 13, of an uncoupled shock followed by a deflagration, and he inquired into the stability of such a system by means of quite a sophisticated analysis of perturbations in transverse coordinates with respect to the plane of the wave front. A somewhat simpler study of the stability of the wave with respect to a two-dimensional disturbance has been proposed by Fay,[37] who developed general perturbation equations for detonation similarity as is done in the perturbation theory of the laminar-boundary-layer instability.

As evidenced by the papers of Zaidel,[106] Shchelkin,[20, 78, 87] Denisov,[16–20] and Troshin,[16–20, 87] the problem of wave stability received a good deal of attention in the USSR. Their arguments are quite hueristic and lead to a simple stability criterion by demanding that the temperature disturbance at the Chapman-Jouguet state J should not reach the von Neumann spike N.

With reference to Fig. 13, the condition for stability is expressed, therefore, simply as follows:

$$|\Delta T\,(dt/dT)| < \tau \tag{1}$$

where

$$t = k\exp(E/RT) \tag{2}$$

where t denotes the time, τ the time lag between states N and J [expressed approximately by (2) with $T = T_N$], T absolute temperature, E activation energy, R the gas constant, and k the reciprocal of the Arrhenius pre-exponential coefficient, taken for this simple theory as a constant. With $\Delta T = T_N - T_J$ and assuming a polytropic relation between pressure and temperature, Eq. (1) with (2) yields the following "Shchelkin stability criterion":

$$(E/RT_N)\,[1 - (p_J/p_N)^{(\gamma-1)/\gamma}] < 1 \tag{3}$$

Since for a typical detonation E/R is of an order of 10^4 °K, whereas T_N is a few thousand degrees and $[1 - (p_J/p_N)^{(\gamma-1)/\gamma}]$ is about 0.1 (for detonation in a stoichiometric hydrogen-oxygen mixture $p_J/p_N = 0.575$, which with $\gamma = 1.25$, yields for the latter exactly 0.105), the foregoing criterion is indeed quite critical, so that, as has already been pointed out on the basis of experimental evidence, for most self-sustained detonations it may not be satisfied at all. This, however, may still be inconsequential, since the instability may lead to finite amplitude oscillations only if its characteristic frequency is in resonance with the whole system.

Structure

Before considering the structure of the reaction zone itself, let us first examine the state of the gas immediately in front of it and immediately behind it. The fact that sharp pressure

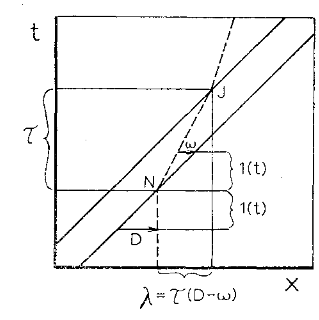

Fig. 13 The "square wave" in the time-space domain.

and density gradients exist at the front of the detonation wave was proposed by Döring, von Neumann, and Zeldovich (see, e. g., Ref. 67 or 96). For detonations propagating into subatmospheric initial prressures, this has been confirmed experimentally by several investigators, notably Kistiakowsky and his students,[13, 47] Wagner and his co-workers,[44, 46, 71, 72, 98] Edwards and his associates,[27–29] White,[56, 100, 101] and Fay.[35, 36]

However, the crucial question, as to whether the so-called von Neumann spike has been attained or not, could not have been answered by these experiments, because their effective space-resolution was still inadequate for such purposes. An extrapolation of the density profile, measured by the x-ray absorption technique,[13, 47] to the front of the reaction zone demonstrated that a peak density of about 70% of the expected von Neumann spike was probably attained. Just and Wagner[45] measured, by means of a schlieren technique, a peak density that was 70 to 90% of the von Neumann spike value. The preliminary records of the pressure profile, obtained by Edwards et al.,[29] can be extrapolated to a pressure peak that seems to be in fair agreement with the von Neumann spike.

The state at the von Neumann spike is usually evaluated assuming complete equilibrium in the different degrees of freedom. It is known in particular that the vibrational relaxation time of several species appearing in the vicinity of the von Neumann spike is of an order of a microsecond. In these cases, therefore, lower peak densities and higher pressures than those of the equilibrium von Neumann state can be expected to occur.

As far as the properties of the gas behind the main reaction zone are concerned, there are some discrepancies between various observations. Although the measurements of Kistiakowsky et al.[13] indicate that, within the experimental accuracy, final densities behind the reaction zone are in agreement with the calculated Chapman-Jouguet state, the more accurate measurements of Duff, Knight, and Rink[23] in mixtures of C_2H_2-O_2, H-$_2O_2$, and C_2N_2-O_2 with krypton demonstrate that the final density is consistently lower than the Chapman-Jouguet value, regardless of whether it is calculated for equilibrium sound speed (when the deviation is about 8%) or frozen (with approximately 3% deviation). The diameters of the tubes used for these experiments were 2.5 and 7.5 cm, tube lengths were 2 to 5 m, and the initial pressure was 20 to 85 cm Hg. Edwards, Williams, and Price[29] measured pressures in hydrogen-oxygen detonations at normal temperature and pressure initial conditions in tubes of different diameters and reported evidence of a definite dependence of the state of the burned gas on the tube size, whereas, surprisingly enough, the detonation velocities were almost constant. White[100] measured pressures in "turbulent detonation waves" propagating in a mixture of $2H_2 + O_2 + 2CO$ at low initial pressures, using a shock tube of 8.5- × 8.5-cm cross section and 13 m long, and obtained data markedly below the calculated Chapman-Jouguet state.

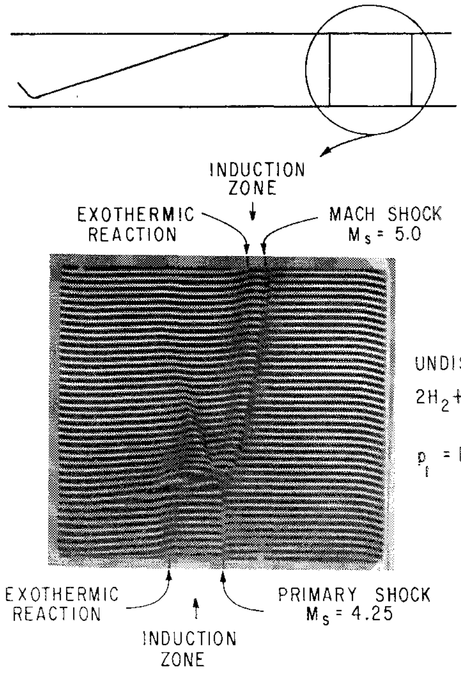

Fig. 14 Interferogram of an overdriven "laminar" detonation in a $2H_2 + O_2 + 2CO$ mixture at an initial pressure of 10 mm Hg, obtained by White[101] in a 8.25-cm square shock tube fitted with a nozzle whose configuration is indicated on sketch above photograph.

Temperatures in the burned gases of detonations were measured by Miyama[59, 60] and Kydd[59] using OH absorption ($C_2H_2 + H_2 + 2O_2$, 60, 120, and 180 mm Hg) and Wagner et al.[45, 95] using OH emission and absorption and line reversal method ($H_2 + O_2$, 150 mm Hg, and 1 atm). Since measurements of OH show large scatter, the line reversal temperatures are considered more accurate. The latter has been found to be essentially in agreement with the calculated values. Finally, an interesting study of the state of explosion products behind shock waves performed on the basis of streak schlieren and pressure transducer records has been made by Tsukhanova.[88]

This brief survey of results shows that a consistent interpretation of the state of the burned gas behind the reaction zone of a detonation is by no means easy and straightforward. Experiments performed in tubes much longer than those used so far should be of great help in clarifying the situation.

Early experiments carried out in order to determine the extent of the reaction zone at atmospheric initial pressures lead to the conclusion that, except for the case of spinning detonations with nodes close to fundamental, the reaction zone was too thin for the measurement of its structure. Consequently, the most indicative measurements in this respect were those performed under subatmospheric initial pressures.

With the use of the x-ray absorption technique, Kistiakowsky et al.[13, 47] were able to obtain some information on density profiles in detonations of Xe-diluted mixtures of H_2-O_2 and CH_4-O_2 and a few others at pressures below 100 mm Hg, and they concluded that the over-all thickness of the main part of the reaction zone is, under such circumstances of an order of 1 cm.

From the measurement of density gradients and OH concentration, Wagner et al.,[44, 45, 71, 98] obtained values of several millimeters for the thickness of the main part of the reaction zone in H_2-O_2 and in some hydrocarbon-oxygen detonations at initial pressures of about 100 mm Hg. These values are supported by measurements of Edwards et al.[29] and Fay and Opel.[34] Jost, Just, and Wagner[44] reported, moreover, the existence of induction periods (zone behind the shock front in which the change in the state of the gas is insignificant) of an order of 1 μsec. This has been confirmed recently by Richmond,[74] who used for this purpose light emission and heat flux measurements, and by Jaarsma and Fuhs,[43] who observed the induction period by recording simultaneously OH radiation and ionization profiles.

Interferograms of self-sustained detonations obtained by White,[100] such as that of Fig. 11, show an irregular shape of the fringes in and behind the reaction zone, whereas the shock front itself is quite smooth and plane. Flash schlieren pictures taken by Fay and Opel[34] demonstrated also that the shape of the reaction zone is irregular. On the basis of optical reflectivity measurements, Sastri, Schwartz, Myers, and Horning[76] concluded that in detonations in $H_2 + 3O_2$ at an initial pressure of 20 psi the wave front is neither plane nor smoothly curved. The irregular shape of the detonation front indeed has been given a lot of attention, as exemplified by the studies of Denisov and Troshin[16–19] and of Duff[25] which already have been described in the previous section, as well as by many other papers that have been published recently in the USSR, such as those of Voitsekhovsky,[93] Mitrofanov,[58, 93] and Topchian.[86, 93]

Although most of these experiments were performed under conditions that did not guarantee the attainment of stable waves, their results seem to support the concept that the reaction zone of a detonation is neither one-dimensional nor smooth, and the question arises as to whether this is always so, and whether such measurements as those of Kistiakowsky[13, 47] and Wagner[96, 98] can be interpreted at all with respect to reaction kinetics. A number of authors expressed an opinion however, that in over-driven detonations the change of state of the gas in the reaction zone can proceed smoothly, as indicated for instance in the interferogram of Fig. 14 which has been obtained recently by White and Cary[101] in a shock tube where the detonation wave was driven through a convergent-divergent nozzle, and the "laminar" wave was evidently established at the stem of a Mach reflection.

In the course of the experimental investigations of Just and Wagner[45] concerning the structure of the reaction zone, many attempts were made to improve the stability and flatness of the waves. It was found then that, in certain mixtures, an increase of the tube length and proper ignition can produce an essentially stable and plane wave, admitting only a possible presence of some kind of spinning mode of propagation with a frequency and number of nodes too high for resolution by the experimental apparatus. In this study spin modes up to the twentieth were observed in H_2-CO-O_2 mixtures. Denisov and Troshin,[16–19] as well as Edwards et al.,[27–29] report still higher modes in H_2-O_2 detonations. With higher modes, however, the spin amplitude decreases, and the disturbance of the shock front, which causes a temperature change in the ignition zone, becomes smaller. Consequently, the fluctuation in the induction period, which is directly dependent on the fluctuation in pressure and temperature, also becomes less intense. A comprehensive theoretical treatment of the reaction zone cannot, of course, be devoid of stability considerations.

The dependence that the extent of the reaction zone can have on the stability can be demonstrated by the following simple argument. With reference to Fig. 13, similarly as for Eq. (2) but with pressure dependence taken into account, the "induction time" can be expressed as

$$\tau = (A/p)\exp(E/RT) \tag{4}$$

where A is a constant and p denotes pressure, all other symbols having the same meaning as before. For a small perturbation Δp in pressure and ΔT in temperature, the corresponding variation in the induction period $\Delta\tau$ is, therefore, to first-order approximation,

$$\frac{\Delta\tau}{\tau} = -\frac{\Delta p}{p} - \frac{E}{R}\frac{\Delta T}{T^2} \tag{5}$$

or, for a polytropic relationship between pressure and temperature,

$$\frac{\Delta\tau}{\tau} = -\left(\frac{\gamma}{\gamma-1} + \frac{E}{RT}\right)\frac{\Delta T}{T} \tag{6}$$

The first term in the parentheses is of an order of five and the second is about eight, so that the variation in the induction period should be more than 10 times larger than the temperature fluctuation.

Incidentally, Eq. (4) may also lead to an interesting interpretation of the wave structure if higher-order terms are not neglected. In the averaging procedure, those that do not cancel out would give rise to the same type of fluctuation term as that used by White[100] to describe the effect of turbulence.

A different approach to the problem, based on the consideration of the boundary layer that causes divergent flow lines behind the shock front, has been given by Fay.[34, 36] A very similar model was treated earlier, but somewhat less extensively, by Döring and Schön.[22]

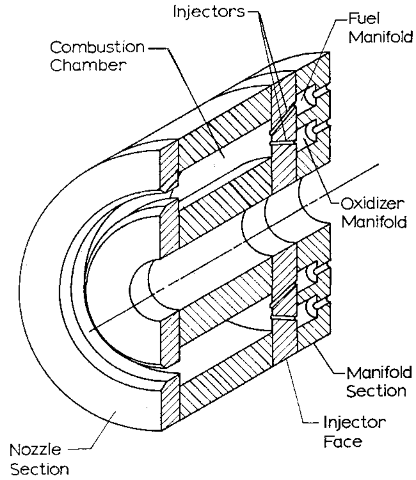

Fig. 15 Rotating detonation wave motor at the University of Michigan.

The two models, "boundary layer" and "turbulence," lead to the same conclusion, namely, that the flow at the "end of the reaction zone" is supersonic with respect to the front of the detonation. This can account for the fact that the pressure and specific volume behind the reaction zone are above their equilibrium Chapman-Jouguet values. A detailed description of the process, however, has not yet been given. Information obtained in tubes that provide sufficiently long travel distance in order to attain smooth, quasi-stationary flow behind the reaction zone should be extremely helpful in this respect. Whether or not the flow at the end of the reaction zone is sonic, whenever this condition is approached, the flow, as it is well known, becomes intrinsically unstable. Perhaps the actual state existing at the "end of the reaction zone" may have to be determined as an average condition through time and cross section.

A good point of departure for such sophisticated inquiries already has been provided by the extensive one-dimensional analyses of the course of chemical reaction in the so-called "laminar" reaction zone which have been carried out by Duff,[24] Hirshfelder and Curtiss,[15, 42, 52] Spalding,[84] Oppenheim and Rosciszewski,[69] and Wood and his associates.[103–105]

Application to Rocketry

The obvious application of the detonation process to rocket propulsion is associated, of course, with schemes for its direct exploitation in the thrust chamber. Especially noteworthy in this respect has been the activity of the group of Morrison and Nicholls at the University of Michigan. Besides an interesting development of a pulsating detonation tube,[64] their investigations led to a significant study of stationary detonation waves.[26, 61–63] Recently, they concentrated upon the possibility of using a rotating detonation wave system for a thrust chamber application.[65] Figure 15 illustrates their experimental thrust chamber where, by a suitable ignition system, a rotating combustion wave resembling detonation is set up in the annulus and propagates azimuthally, whereas the main flow of reactants and products is in the axial direction. Interestingly enough, quite independently, a similar system has been employed at the same time by Voitsekhovsky[89–92] in Novosibirsk as an attempt to attain a steady detonation wave process.

Of most profound importance to rocket technology is the facility that the detonation tube provides for the study of the combustion process in the presence of a pressure gradient, in particular with reference to its stability. It is sufficient to consider the similarities in thermodynamic operating conditions and in the power density of heat release which is achieved in the two systems in order to realize the usefulness of detonation research to rocket technology. It is of interest to note, for instance, that a rocket thrust chamber having a combustion volume of approximately 14 ft^3 and delivering about 1.5×10^6 lb of thrust at an expenditure of some 2000 lb/sec of fuel (based on Ref. 108) generates about 4×10^7 Btu/sec of heat power or operates at a power density level of 100 mw/liter. Exactly the same level is obtained in a stoichiometric hydrogen-oxygen detonation propagating into an initial pressure of 0.1 atm, whereas for atmospheric initial pressure the wave develops heat power at a density of 10^4 Mw/liter.

Except for some sporadic instances, such as the application of the detonation wave to the study of spray combustion which has been carried out by Webber[99] and Cramer[14] experimentally and by Williams[102] analytically, similarity between the two systems has been so far somewhat underestimated in our countries, whereas our Russian colleagues have evidently devoted a good deal of attention to it for quite some time (see, e.g., Refs. 87, 78, and 20). Denisov, Shchelkin, and Troshin[20] show how the stability criterion for a detonation wave can be applied to a rocket thrust chamber.

The "Shchelkin criterion" has essentially the same form as in Eq. (3), except that subscript N denotes now the state of the unburned mixture, immediately downstream of the showerhead, and, instead of p_J, one has only $p = p_N - \Delta p$, where Δp is the pressure perturbation. For a small value of Δp, the criterion then becomes

$$\frac{\gamma - 1}{\gamma} \frac{E}{RT_N} \frac{\Delta p}{p_N} < 1 \qquad (7)$$

Pressure change Δp can be related to the flow Mach number M and heat release per unit mass q by considering it as one associated with a process at constant stream force per unit area. This is represented by a Rayleigh line whose intersection with a Hugoniot curve gives the following approximate expression:

$$\Delta p/p = \gamma(\gamma - 1)\, M^2\, (q/a^2) \qquad (8)$$

where a is the local velocity of sound. Equation (7) then reduces to the following form given by Denisov et al.[20]:

$$(\gamma - 1)^2\, (E/RT_N)\, M_N{}^2\, (q/a_N{}^2) < 1 \qquad (9)$$

This can be further simplified without any loss in generality by noting that

$$M/a = \dot{m}/\gamma p \qquad (10)$$

where $\dot{m}$ is the mass flow rate per unit area. The "Shchelkin criterion" for the rocket thrust chamber then reduces finally to the following stability condition:

$$\left(\frac{\gamma - 1}{\gamma}\right)^2 \frac{\dot{m}^2\, (E/R)\, q}{P_N{}^2\, T_N} < 1 \qquad (11)$$

For a representative case of $\gamma = 1.25$, $\dot{m} = 1000$ lb/ft^2-sec, $E/R = 20{,}000$°R, $q = 4000$ Btu/lb, $p_N = 500$ psi, and $T_N = 1000$°R, the criterion has a value of 15, indicating indeed a condition of an intrinsic instability.

Such an over-all concept as the foregoing is handicapped, of course, by an oversimplification in the formulation of the problem. A possibility of existence is by no means a sufficient condition for the occurrence of unstable operation. Basically, an instability analysis should be concerned with two problems: 1) the sustenance of the deviations from equilibrium which are usually manifested by finite amplitude oscillations, and 2) the driving mechanism.

Most of the theories on combustion instability in rocket thrust chambers are concerned with some aspects of the former, primarily, in fact, with the determination of the regimes of operating conditions under which the deviations can be sustained. The "Shchelkin criterion" represents a simplified example of such an approach.

As to the latter, so far it has received indeed very little attention. Since, as a rule, an unstable operation is associated with pressure waves, it is the study of the generation of such waves accompanying the release of heat, as illustrated here, for instance, with the observations of accelerating flames, which should be quite revealing in this respect.

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